Activity

Topic
Similarity of Two Triangles

Objective
To establish the criteria for similarity of two triangles.

Material Required
Coloured papers, glue, sketch pen, cutter, geometry box.

Method of Construction I

1. Take a coloured paper/chart paper. Cut out two triangles ABC and PQR with their corresponding angles equal.

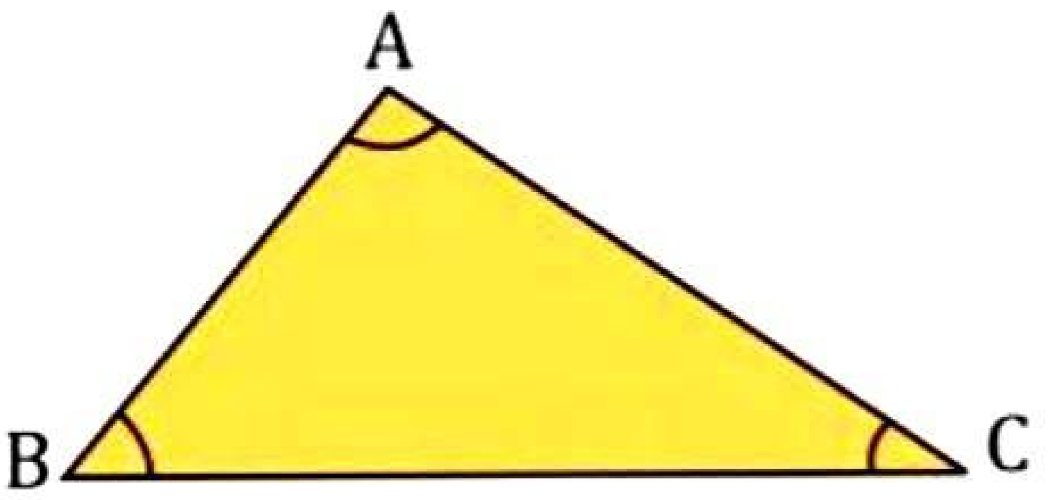

Fig.1

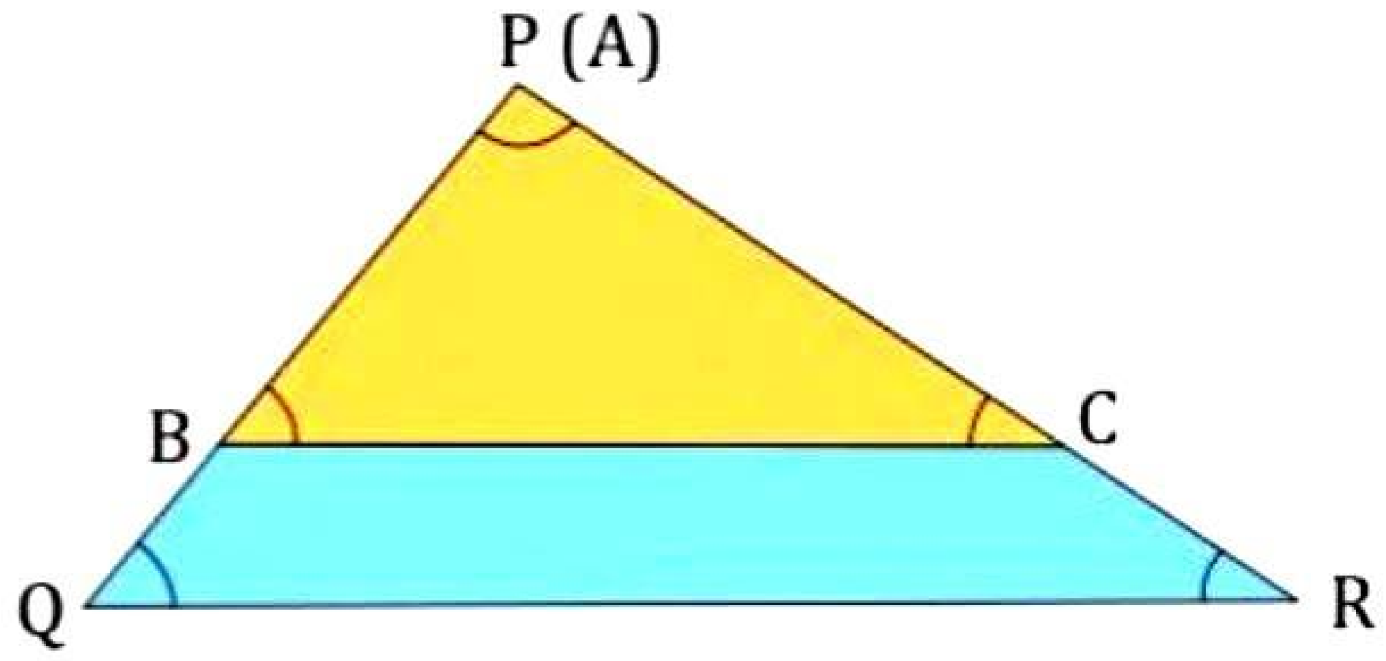

Fig.2

2. In the triangles ABC and PQR, $\angle A = \angle P$; $\angle B = \angle Q$ and $\angle C = \angle R$.
3. Place the $\triangle$ ABC on $\triangle$ PQR such that vertex A falls on vertex P and side AB falls along side PQ (side AC falls along side PR) as shown in Fig. 2.

Demonstration I

In Fig. 2, $\angle B = \angle Q$. Since corresponding angles are equal, BC ∥ QR

By BPT,

$$\frac{PB}{BQ} = \frac{PC}{CR} \text{ or } \frac{AB}{BQ} = \frac{AC}{CR} \text{ or } \frac{BQ}{AB} = \frac{CR}{AC}$$

$$\text{or} \quad \frac{BQ + AB}{AB} = \frac{CR + AC}{AC} \quad \text{[Adding 1 to both sides]}$$

$$\text{or} \quad \frac{AQ}{AB} = \frac{AR}{AC} \text{ or } \frac{PQ}{AB} = \frac{PR}{AC} \text{ or } \frac{AB}{PQ} = \frac{AC}{PR} \text{----} (1)$$

Method of Construction II

1. Place the $\triangle$ ABC on $\triangle$ PQR such that vertex B falls on vertex Q, and side BA falls along side QP (side BC falls along side QR) as shown in Fig. 3.

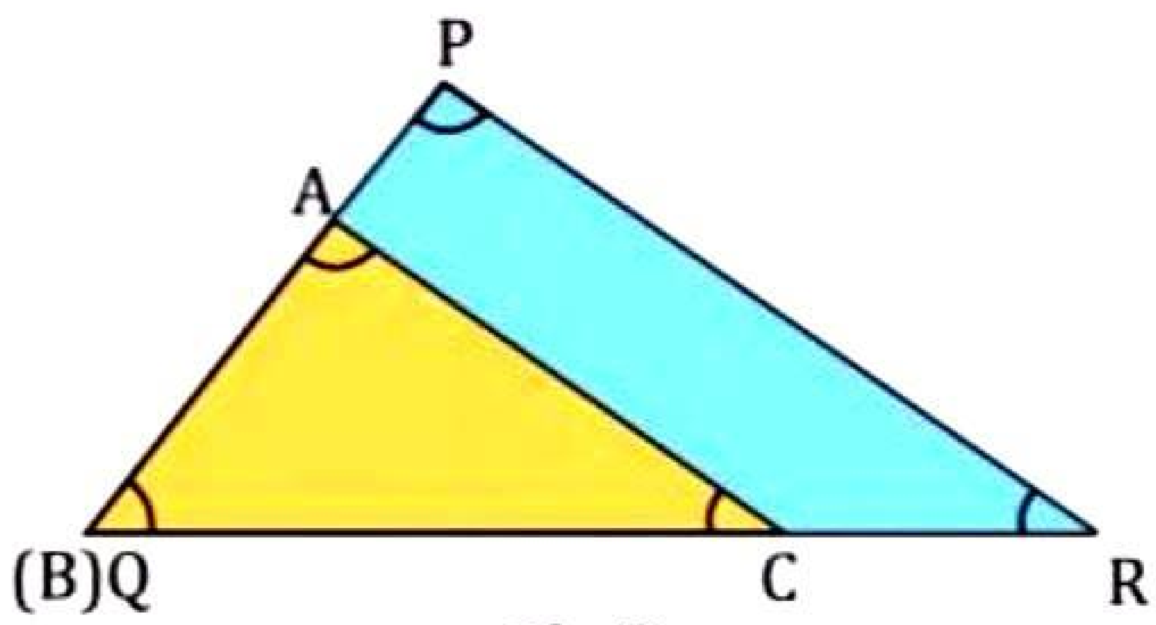

Fig.3

Demonstration II

In Fig. 3, $\angle C = \angle R$. Since corresponding angles are equal, $AC \parallel PR$

By BPT, $\frac{AP}{AB} = \frac{CR}{BC}$; or $\frac{BP}{AB} = \frac{BR}{BC}$ [Adding 1 on both sides]

or $\frac{PQ}{AB} = \frac{QR}{BC}$ or $\frac{AB}{PQ} = \frac{BC}{QR}$......(2)

From (1) and (2), $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

Thus, from Demonstrations I and II, we find that when the corresponding angles of two triangles are equal, then their corresponding sides are proportional. Hence, the two triangles are similar. This is AAA criterion for similarity of triangles.

Alternatively, you could have measured the sides of the triangle's ABC and PQR and obtained.

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$.

From this Result, △ ABC and △ PQR are similar, i.e., if three corresponding angles are equal, the corresponding sides are proportional and hence the triangles are similar. This gives AAA criterion for similarity of two triangles.

Method Of Construction III

1. Take a coloured paper/chart paper, cut out two triangles ABC and PQR with their corresponding sides proportional.
2. i.e., $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

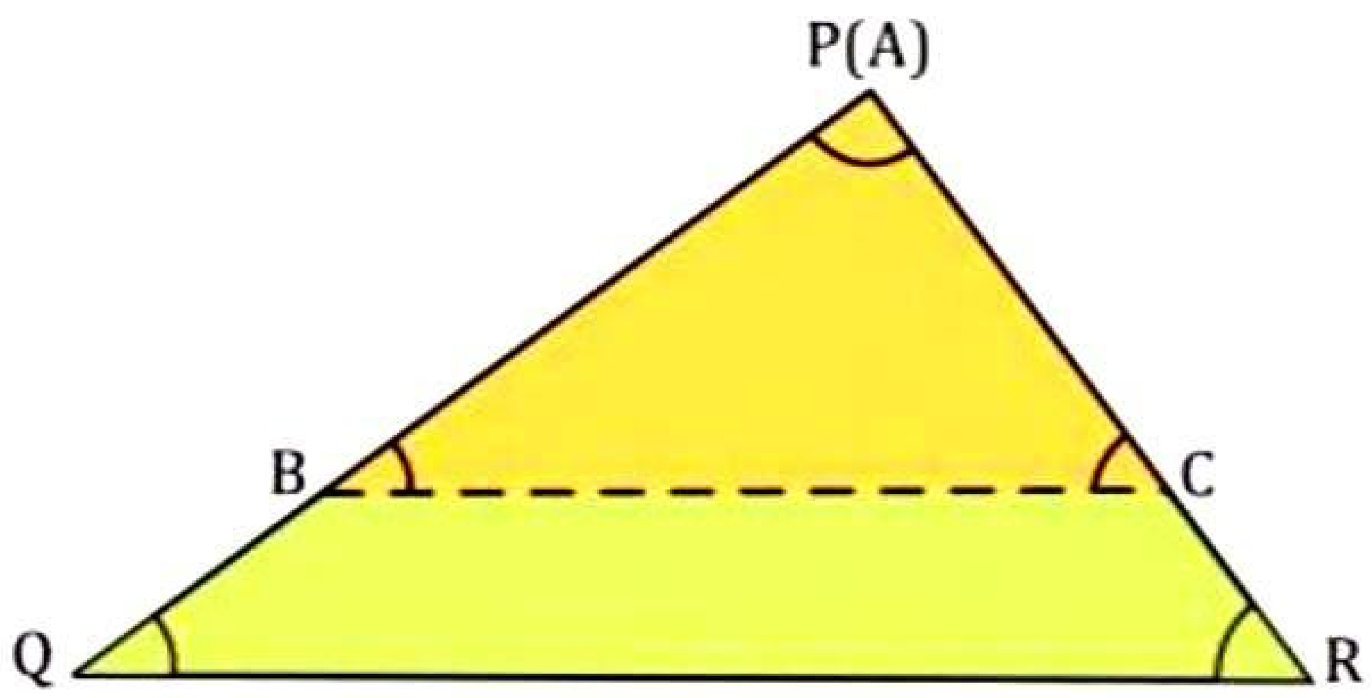

Fig.4

3. Place the △ ABC on △ PQR such that vertex A falls on vertex P and side AB falls along side PQ. Observe that side AC falls along side PR [see Fig. 4].

Demonstration III

In Fig. 4, $\frac{AB}{PQ} = \frac{AC}{PR}$. This gives $\frac{AB}{BQ} = \frac{AC}{CR}$. So, BC ∥ QR (by converse of BPT) i.e., $\angle B = \angle Q$ and $\angle C = \angle R$. Also $\angle A = \angle P$. That is, the corresponding angles of the two triangles are equal.

Thus, when the corresponding sides of two triangles are proportional, their corresponding angles are equal. Hence, the two triangles are similar. This is the SSS criterion for similarity of two triangles.
Alternatively, you could have measured the angles of $\triangle$ ABC and $\triangle$ PQR and obtained $\angle A = \angle P, \angle B = \angle Q$ and $\angle C = \angle R$.

From this Result, $\triangle$ ABC and $\triangle$ PQR are similar, i.e. if three corresponding sides of two triangles are proportional, the corresponding angles are equal, and hence the triangles are similar. This gives SSS criterion for similarity of two triangles.

Method of Construction IV

1. Take a coloured paper/chart paper, cut out two triangles ABC and PQR such that their one pair of sides is proportional and the angles included between the pair of sides are equal.

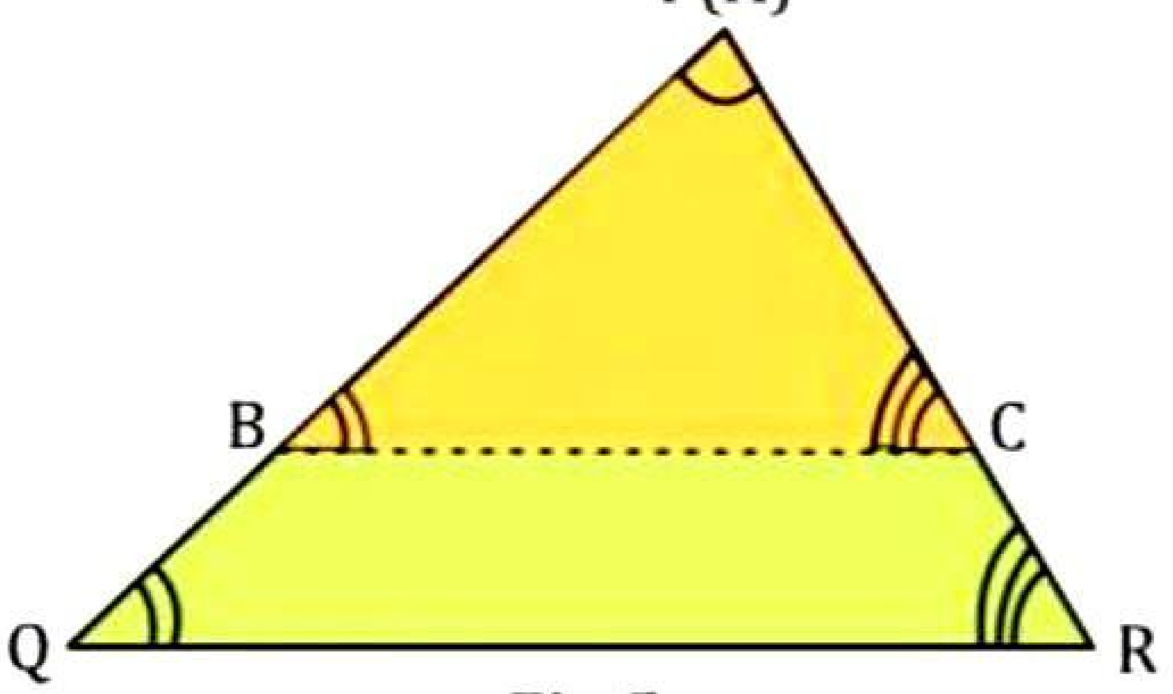

Fig.5

i.e., In $\triangle$ ABC and $\triangle$ PQR, $\frac{AB}{PQ} = \frac{AC}{PR}$ and $\angle A = \angle P$.

2. Place triangle ABC on triangle PQR such that vertex A falls on vertex P and side AB falls along side PQ as shown in Fig. 5.

Demonstration IV

In Fig. 5, $\frac{AB}{PQ} = \frac{AC}{PR}$. This gives $\frac{AB}{BQ} = \frac{AC}{CR}$. So, BC||QR (by converse of BPT)
Therefore, $\angle B = \angle Q$ and $\angle C = \angle R$.
From this demonstration, we find that when two sides of one triangle are proportional to two sides of another triangle and the angles included between the two pairs of sides are equal, then corresponding angles of two triangles are equal.
Hence, the two triangles are similar. This is the SAS criterion for similarity of two triangles.
Alternatively, you could have measured the remaining sides and angles of $\triangle$ ABC and $\triangle$ PQR and obtained $\angle B = \angle Q, \angle C = \angle R$ and $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$. From this, $\triangle$ ABC and $\triangle$ PQR are similar and hence we obtain SAS criterion for similarity of two triangles.

Observation

By actual measurement:

I. In $\triangle$ ABC and $\triangle$ PQR,

$\angle A = \cdots, \angle P = \cdots, \angle B = \cdots, \angle Q = \cdots, \angle C =$
$\angle R = \cdots,$

$\frac{AB}{PQ} =$ ______; $\frac{BC}{PR} =$ ______; $\frac{AC}{PR} =$ ______

If corresponding angles of two triangles are ______, the sides are ______. Hence the triangles are ______.

II. In △ ABC and △ PQR

$\frac{AB}{PQ}$ = ________; $\frac{BC}{QR}$ = ________; $\frac{AC}{PR}$ = _____

∠A = __________, ∠B = _______, ∠C = ________. ∠P = _________,

∠Q = __________, ∠R = ___________,

If the corresponding sides of two triangles are then their corresponding angles are _______. Hence, the triangles are __________.

III. In △ ABC and △ PQR,

$\frac{AB}{PQ}$ = _____; $\frac{AC}{PR}$ = ____

∠A = ____, ∠P = ______, ∠B = ____

∠Q = ______

∠C = ______

∠R = ______

If two sides of one triangle are ___ to the two sides of other triangle and angles included between them are ___, then the triangles are ________.

Application

The concept of similarity is useful in reducing or enlarging images or pictures of objects.

VIVA VOCE

Q 1. Define similarity of triangles.

Ans. Similarity of triangles refers to a relationship between two triangles in which their corresponding angles are equal, and the corresponding sides are in proportion.

Q 2. What is the Angle-Angle (AA) criterion for similarity?

Ans. The Angle-Angle (AA) criterion states that if two triangles have two corresponding angles that are equal, then the triangles are similar.

Q 3. Can two triangles be similar if their corresponding sides are proportional but the corresponding angles are not equal?

Ans. No, for triangles to be considered similar, it is necessary that their corresponding angles are equal. Proportional sides alone are not sufficient for similarity.

Q 4. How does the Side-Angle-Side (SAS) criterion contribute to proving the similarity of two triangles?

Ans. The Side-Angle-Side (SAS) criterion states that if two triangles have a pair of corresponding sides that are in proportion and the included angles are equal, then the triangles are similar.

Q 5. Can the Side-Side-Angle (SSA) criterion alone guarantee the similarity of two triangles?

Ans. No, the Side-Side-Angle (SSA) criterion alone is not sufficient to guarantee the similarity of two triangles. Additional information about the proportionality of corresponding sides is needed for a conclusive similarity proof.

MULTIPLE CHOICE QUESTIONS

Q 1. Two triangles are similar if:

(a) They have equal angles.
(b) Their corresponding angles are congruent.
(c) Their corresponding sides are proportional.
(d) They have the same perimeter.

Q 2. In similar triangles, the ratio of the corresponding angles is:

(a) Equal (b) Greater than 1 (c) Proportional (d) Constant

Q 3. If two angles of one triangle are equal to two angles of another triangle, then the triangles are:

(a) Similar (b) Congruent
(c) Neither similar nor congruent (d) Inversely proportional

Q 4. The AAA (Angle-Angle-Angle) criterion for similarity states that:

(a) Two triangles are similar if their corresponding angles are congruent.
(b) Two triangles are similar if their corresponding sides are proportional.
(c) Two triangles are similar if they have the same area.
(d) Two triangles are similar if they have a common side.

Q 5. If the lengths of the corresponding sides of two triangles are in the ratio 3:5, and their perimeters are 24 cm and 40 cm respectively, then the lengths of the corresponding sides are:

(a) 6 cm and 10 cm (b) 9 cm and 15 cm
(c) 12 cm and 20 cm (d) 15 cm and 25 cm

Answer Key

1.(c)	2.(a)	3.(a)	4.(a)	5.(b)